### C-89 Sphere Surface Area Conjecture 10.7

The surface area of a sphere is given by the equation...

$$SA = 4\pi r^2$$

**Ex.**

$$SA = 4\pi (4^2) = 64\pi \text{ in}^2 \approx 201.0619 \text{ in}^2$$

Be careful on 3A of a hemisphere!!

$$SA = \frac{1}{2} 4\pi r^2 + \pi r^2 \text{ for the bottom}$$

$$= 3\pi r^2 + 16\pi r^2 = 19\pi r^2 \approx 150.7964 \text{ in}^2$$

### C-90 Dilation Similarity Conjecture 11.1

If one polygon is a dilated image of another polygon, then the polygons are similar.

### C-91 AA Similarity Conjecture 11.2

If 2 angles of one Δ are \( \cong \) to 2 Δs of another Δ, then the Δs are similar.

### Algebra Review 11: Solving Proportions

Use a cross-multiply technique based on the algebra that you multiply both sides by denom.

Use distributive property for (quantities) in numer. or denom.

**Basic Ex.**

\[
\frac{5}{8} = \frac{14.5}{x} \\
x = \frac{56}{5}
\]

**Complex Ex.**

\[
\frac{x+14}{20} = \frac{x+32}{100} \\
20(x+14) = 60(x+32) \\
20x + 280 = 60x + 960 \\
-40x = 680 \\
x = -17
\]

### Trap. SMAL is 50% (or \( \frac{1}{2} \)) the size of Trap BIER \( \sim \) SMAL \( \sim \) BIER blc sides maintain same ratio and L's are all \( \cong \).
### SSS Similarity Conjecture 11.2

**If all 3 sides of one \( \triangle \) are proportional to the 3 sides of another \( \triangle \) then the 2 \( \triangle \)'s are **similar**!!**

<table>
<thead>
<tr>
<th>A</th>
<th>2.2 cm</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>2 cm</td>
<td>E</td>
</tr>
<tr>
<td>2.4 cm</td>
<td>2 cm</td>
<td></td>
</tr>
</tbody>
</table>

\[ \frac{B}{C} = \frac{2}{6} = \frac{2.2}{6.6} = \frac{2.4}{7.2} = \frac{1}{3} \] common ratio

\[ \therefore \triangle AMK \sim \triangle EWN \text{ by SSS similarity} \]

### SAS Similarity Conjecture 11.2

**If 2 sides of 1 \( \triangle \) are proportional to 2 sides of another \( \triangle \) and the included \( \angle \)s are \( \cong \) then the \( \triangle \)'s are **similar**!!**

\[ \triangle ALH \sim \triangle FTB \text{ by SAS similarity} \]

- \( \frac{H}{L} = \frac{2}{3} \]
- \( \frac{F}{T} = \frac{2}{6} = \frac{1}{3} \text{ common ratio} \]
- \( \angle L \cong \angle T \)

### Proportional Parts Conjecture 11.4

**If 2 \( \triangle \)'s are \( \cong \), then the lengths of the corresponding altitudes, medians, and \( \angle \) bisectors are proportional to the lengths of the corresponding sides.**

\[ \frac{a}{b} = \frac{c}{d} = \frac{3}{6} = \frac{1}{2} \text{ for side ratios} \]

### Angle Bisector/Opposite Side Conjecture 11.4

**A bisector of an angle in a \( \triangle \) divides the opposite side into 2 segments whose lengths are in the same ratio as the lengths of the two sides forming the angle.**

\[ \frac{a}{b} = \frac{c}{d} \text{ same ratios} \]
### C-96 Proportional Areas Conjecture 11.5

If corresponding side lengths of 2 similar polygons or the radii of 2 circles compare in the ratio \( m/n \), then their areas compare in the ratio \( m^2/n^2 \).

**Example:** If two similar figures have a side ratio of 2:5, and the larger figure has an area of 115 cm², then what is the area of the smaller?

**Answer:**

\[
\frac{m}{n} = \frac{2}{5} \quad \Rightarrow \quad \frac{4}{25} = \frac{x}{115} \quad \Rightarrow \quad 25x = 460 \quad \Rightarrow \quad x = 18.4 \text{ cm}^2
\]

### C-97 Proportional Volumes Conjecture 11.6

If corresponding edge lengths (or radii or heights, etc...) of 2 similar solids compare in the ratio \( m/n \), then their volumes compare in the ratio \( m^3/n^3 \).

**Example:** The prisms are similar.

[Diagram of two prisms with measurements]

**Answer:**

\[
\frac{m}{n} = \frac{3}{5} \quad \Rightarrow \quad \frac{27}{125} = \frac{x}{210} \quad \Rightarrow \quad 27x = 26,250 \quad \Rightarrow \quad x = 972.2 \text{ in}^3
\]

### C-98 Parallel/Proportionality Conjecture 11.7

If a line \( \parallel \) to one side of a \( \triangle \) passes thru the other two sides, then it divides the other 2 sides proportionally.

*Conversely, if a line cuts 2 sides of a \( \triangle \) proportionally, then it is parallel to the third side.*

[Diagram of triangle with parallel line cutting sides]

**Lots of Ratios you can write...**

\[
\frac{a}{b} = \frac{e}{d} \quad \frac{a+b}{e+d} = \frac{c}{f} \quad \frac{a+b}{c} = \frac{d}{f} \quad \frac{a}{b+tc} = \frac{d}{ef}
\]

### C-99 Extended Parallel/Proportionality Conjecture 11.7

If two or more lines pass thru a sides of a \( \triangle \parallel \) to the third side, then they divide the two sides proportionally.

[Diagram of triangle with multiple parallel lines cutting sides]
<table>
<thead>
<tr>
<th>Valid Forms of Reasoning CH 11 Exploration</th>
<th>SOH CAH TOA Ratios 12.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CONVERSE</strong>: If ( p ) then ( q ) ( \rightarrow ) If ( q ) then ( p )</td>
<td>SOH CAH TOA is a memory device to remember trigonometry ratios.</td>
</tr>
<tr>
<td><strong>INVERSE</strong>: If ( \sim p ) then ( \sim q )</td>
<td>( \sin \theta = \text{opposite} )</td>
</tr>
<tr>
<td><strong>CONTRAPPOSITIVE</strong>: If ( \sim q ) then ( \sim p )</td>
<td>( \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} ) Only works for right triangles!!</td>
</tr>
<tr>
<td>ex) If you are at a movie then you are not at Starbucks.</td>
<td>( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} )</td>
</tr>
<tr>
<td><strong>CONVERSE</strong>: If you are not at Starbucks then you are at a movie.</td>
<td>( \theta ) is part of the Greek alphabet. It is commonly used for missing angles.</td>
</tr>
<tr>
<td><strong>INVERSE</strong>: If you are not at a movie then you are at Starbucks</td>
<td>Finding Missing Sides w/ SOH CAH TOA 12.1</td>
</tr>
<tr>
<td><strong>CONTRAPPOSITVE</strong>: If you are at Starbucks then you are not at a movie.</td>
<td>Finding Missing Angles w/ SOH CAH TOA 12.1</td>
</tr>
<tr>
<td>Always same as original F &amp; F</td>
<td>By using trig ratios, you can find a missing side if you only have one side &amp; one ( \angle ).</td>
</tr>
</tbody>
</table>

In a Right \( \triangle \)

Ex. 1)

\[
\sin 26^\circ = \frac{X}{50}
\]

\[50 \cdot \sin 26^\circ = X\]

\[X \approx 21.9185 \text{ cm}\]

Ex. 2)

\[
\cos 33^\circ = \frac{40}{X}
\]

\[X \cdot \cos 33^\circ = 40\]

\[X = \frac{40}{\cos 33^\circ}\]

\[X \approx 33.5468 \text{ cm}\]

\[
\tan \theta = \frac{13}{15}
\]

\[\theta = \tan^{-1} \left(\frac{13}{15}\right)\]

\[\theta \approx 40.9143^\circ\]